Optimization of Large-Scale Rigidified Inflatable Structures using Physical Programming

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Bibliographical Information
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Abstract  This paper makes important initial steps in the application of large-scale structural optimization to Rigidified Inflatable Structures (RIS) for cost competitive residential construction, and does so within the realistic framework of multiobjective optimization using the effective physical programming approach. Over the past two decades, structural optimization has proved to be an invaluable tool in numerous arenas. Its faint beginnings in civil engineering have given way to important applications in the aerospace industry, and more recently, in the automotive industry and many other areas. Importantly, structural optimization has given way to the broader field of Multidisciplinary Design Optimization (MDO). Within this context, this paper explores the feasibility of RIS design for residential construction with respect to cost, structural integrity (e.g., buckling, deformation), and other practical issues. A cylindrical structure is considered, and is subjected to code-specified snow and wind loads. Within a multiobjective framework, a physical-programming-based optimization approach is developed to examine the behavior and feasibility of reinforced and non-reinforced polymers as primary RIS materials. Using a finite element model of approximately 72,000 degrees of freedom, we illustrate how the physical programming method effectively addresses the multiobjective and multiscale nature of the problem. Initial results indicate favorable feasibility of RIS use in housing. Further studies of broader scope are suggested.

Key words  rigidified inflatable structures, multiobjective optimization, structural optimization, physical programming, housing

1 Introduction

Residential Construction: The residential construction industry is widely reputed to be highly conservative – with revolutionary innovations as rare occurrences. However, there are many aspects of residential construction, and of the construction industry in general, that can benefit from recent technological developments. (i) One such aspect is the large amount of resources the construction industry consumes. Almost one half of the world’s major resources are consumed by construction and related industries, and fifty-five percent of the wood cut for non-fuel purposes is used in construction (Roodman and Lenssen 1995). In light of this observation, the development of alternative construction systems that have reduced material usage at a competitive cost is important. (ii) In addition, the construction time and cost of conventional housing does not readily lend itself to emergency situations. Lower cost and rapid construction technologies would likely find broad use. Rigidified Inflated Structures (RIS) offers significant potential with regard to the above observations.

Rigidified Inflatable Structures for Residential Construction: Rigidified Inflatable Structures are thin flexible membranes that – when pneumatically deployed – rigidify into substantially strong, load-bearing thin-shell structures. The development of RIS originated in the space industry, where its use has been demonstrated in applications such as large solar arrays (Malone and Williams 1996) and large parabolic antennas (Freeland et al. 1993).

Although the residential construction industry differs significantly from the space industry, RIS technology has been previously suggested for residential construction (Dent 1972; DiTomas 1996). Indeed, there are appealing benefits to using RIS in the traditional construction industry, including reduced material usage, relatively simple installation, and potential for reduced costs. The use of RIS in residential construction is an innovative idea that could revolutionize certain aspects of the construction industry. Previous work of more limited scope (Van Dessel et al. 2002a,b) in this area has investigated the feasibility of RIS systems for use in residential construc-
tion. The path that lies ahead to infuse RIS into residential construction includes the optimization of material properties that result in structures that achieve reduced material usage, high load bearing capability, and low costs.

**Structural Optimization:** The present study is cast as a structural optimization problem. We focus here on two important aspects of structural optimization. First, structural optimization is an inherently multiobjective problem (e.g., minimize cost, maximize load capacity, minimize stress). Second, large structural optimization problems often yield a complex set of governing equations that result in objectives that involve several scales, particularly when considered in conjunction with other non-structural objectives. In addition, this multiobjective and multi-scale environment often gives rise to non-convex Pareto frontiers. As such, conventional weighted sum Aggregate Objective Functions (AOF) are distinctly ineffective. Furthermore, numerical issues associated with the different scales often present significant difficulties in the application of other more powerful methods, such as compromise programming. We therefore took special care to employ a multiobjective optimization tool that has the ability to optimize large-scale problems in an effective environment. The choice of physical programming was chiefly guided by these considerations.

**Structural Optimization for Large-Scale Structures:** In the automotive and aerospace industries, as well as in numerous other fields, structural optimization of large structures or large-scale systems is a typically successful undertaking. Various computational techniques have contributed to the current state of the art. Since a comprehensive presentation of structural optimization is not the object of this paper, we shall limit our comments to aspects of structural optimization that are pertinent to the evolution of RIS technology. Chen et al. (2001) developed a parallel nodal-based evolutionary structural optimization method in which concurrent optimization is performed on the interior and exterior of the domain, using topology and shape optimization, respectively. In their method, the element distortions are corrected by iteratively performing finite element analyses and re-meshing. The genetic algorithm (GA) is a popular approach used to optimize large-scale structures. This method requires a large number of function evaluations, which is a disadvantage that may be offset when each function evaluation is very fast – possibly because of the use of approximation techniques. Reviews of these and related methods are abundant in the literature (Vanderplaats 1999). Jenkins (1997) discusses special precautions required to adapt GAs to large-scale applications. The adapted technique uses space condensation heuristics and adaptivity of controls to assist GAs in performing optimization for large-scale structural problems. The amount of time required to optimize such large-scale structures using GA-based techniques is enormous, which motivates researchers (Adeli and Kumar 1995) to develop efficient parallel algorithms for the optimization of large structures using massively parallel supercomputers. Another approach for optimizing large-scale structures was developed by Patnaik and Hopkins (2000). Their method uses the sub-structuring strategy of the general purpose optimization tool COMETBOARDS, which divides the large structure into smaller substructures, and performs repeated optimizations on those substructures.

**Multiobjective Structural Optimization:** The most prevalently used multiobjective structural optimization algorithms include such methods as compromise programming, goal programming, physical programming, modified game theory, and surrogate worth trade-off. Sunar and Rao (1995) presented a basic multiobjective problem formulation for the design of flexible structures by using goal programming and modified game theory. Another multiobjective structural optimization problem, which was presented by Tseng and Lu (1990), addresses the selection of system parameters and large-scale structural design optimization, and is solved by adapting three distinct methods: goal programming, compromise programming, and the surrogate worth trade-off method. Rao (1994) presented a solution procedure for solving a general multiobjective structural optimization problem. The method combines the concepts of fuzzy set theory and game theory, and is applied to the optimization of actively controlled structures.

**Physical Programming for Structural Optimization:** Physical Programming (PP) is a multiobjective optimization tool that explicitly incorporates the Decision Maker (DM) preferences with respect to each design objective into the optimization process (Messac 1996). It has the advantage of guiding DMs through a relatively simple process of forming a suitable AOF without the typical iterative process involving physically meaningless weights, which is required by virtually all other multiobjective optimization tools. Another significant trait of PP that facilitates its use in structural optimization is its ability to effectively optimize badly scaled problems. An example of such problems in structural optimization is the minimization of two quantities of different magnitudes, such as the minimization of stress (on the order of $10^7$) and deflection (on the order of $10^{-2}$). Previous research (Messac et al. 1996; Messac and Hattis 1996; Messac and Wilson 1998; Wilson et al. 1999; Suleman and Goncalves 2000), which has concentrated on medium scale problems, illustrates the effectiveness of PP in addressing a wide array of multiobjective problems. In the present application, we apply physical programming to a larger problem (i.e., the applications stated above). As will be shown, physical programming offers a highly effective means of exploring the feasibility of RIS applications – through optimization.

**Scope of Study:** As stated earlier, this study aims to break ground in exploring the feasibility of RIS technology for residential housing. The scope of this study involves distinct components. (i) This study involves the development of a generic structure that is used as a testbed for answering the primordial question “Can RIS sustain expected structural loads?” (ii) This study
establishes the feasibility of various classes of RIS material properties. (iii) This study develops a multiobjective structural analysis and optimization methodology for large-scale problems based on Physical Programming. (iv) This study employs the structural optimization program Genesis as the structural analysis and optimization engine, with which physical programming is mated to perform the system optimization. We note that this study does not address other important aspects of the application of RIS technology to housing. In particular, our structural testbed does not include windows and doors, nor does it include such accessories as electrical and plumbing materials. In addition, in certain cases it may be desirable to include filling materials within the hollow rigidified structure. We have not considered such possibilities. We have also not modeled the actual deployment process, nor have we addressed aerodynamic issues. We have instead focused on addressing the most important issues; and have done so within the context of multiobjective structural optimization – as a means of offering new possibilities in the expected evolution of RIS technology.

The remainder of the paper is organized as follows. A technical synopsis is provided in Sect. 2, followed by the RIS modeling approach in Sect. 3, and the optimization problem formulation and approach in Sect. 4. Results and discussion are presented in Sect. 5, and Sect. 6 concludes the paper.

2 Technological foundations

This section reviews the two foundational technologies that form the basis of the development of this paper: (i) Physical Programming and (ii) Rigidified Inflatable Structures.

2.1 Physical programming synopsis

Physical Programming (Messac 1996) is an optimization method that has the capability of effectively optimizing multiobjective problems. The method capitalizes on the availability of decision-maker design knowledge and preferences, and utilizes these to guide the optimization process. Within the physical programming procedure, the Decision Maker (DM) expresses his or her preferences with respect to each design metric using four different classes by declaring that each belongs to one of the classes. Each class comprises two cases, hard and soft, referring to the sharpness of the preference. Figure 1 depicts the qualitative and quantitative meanings of each soft class. The value of the criterion under consideration, \( \mu_i \), is shown on the horizontal axis, and the function that will be minimized for that criterion, \( P_i \), hereby called the class-function, is shown on the vertical axis. All soft class functions will become constituent components of the aggregate objective function.

Physical programming allows the user to express preferences with regard to each criterion with more specificity and flexibility than by simply saying minimize, maximize, greater than, less than, or equal to. The preferences are characterized by degrees of desirability as seen in Fig. 1. Consider, for example, the case of Class 1S (topmost plot of Fig. 1). The preference ranges are:

- **Highly Desirable range** \( (\mu_i \leq \nu_{i1}) \)
- **Desirable range** \( (\nu_{i1} \leq \mu_i \leq \nu_{i2}) \)
- **Tolerable range** \( (\nu_{i2} \leq \mu_i \leq \nu_{i3}) \)
- **Undesirable range** \( (\nu_{i3} \leq \mu_i \leq \nu_{i4}) \)
- **Highly Undesirable range** \( (\nu_{i4} \leq \mu_i \leq \nu_{i5}) \)
- **Unacceptable range** \( (\mu_i \geq \nu_{i5}) \)

The parameters \( \nu_{i1} \) through \( \nu_{i5} \) are physically meaningful constants that express the DM’s preference associated with the \( i \)th generic design metric.
Class-functions are used to map design metrics into non-dimensional, strictly positive, real numbers. This mapping, in effect, transforms design metrics with disparate units and physical meaning onto a dimensionless scale through a unimodal function. Consider the first segment of the curve for Class 1S in Fig. 1. When the value of the criterion, \( \mu_i \), is less than \( \nu_{i3} \) (Highly Desirable range), the value of the class function is small, and requires little further minimization. If, on the other hand, the value of the metric, \( \mu_i \), is between \( \nu_{i4} \) and \( \nu_{i5} \) (Highly Undesirable range), the value of the class function is large, and necessitates significant minimization. Stated simply, the value of the class-function for each design metric governs the optimization path in objective space.

The physical programming problem model then takes the following form:

\[
\min \ P(\mu) = \frac{1}{n_s} \sum_{i=1}^{n_s} P_i(\mu_i(x))
\]

subject to

\[
\mu_i(x) \leq \nu_{i5} \quad \text{(for Class 1S objectives)}
\]

\[
\mu_i(x) \geq \nu_{i5} \quad \text{(for Class 2S objectives)}
\]

\[
\nu_{i5L} \leq \mu_i(x) \leq \nu_{i5R} \quad \text{(for Class 3S objectives)}
\]

\[
\nu_{i5L} \leq \mu_i(x) \leq \nu_{i5R} \quad \text{(for Class 4S objectives)}
\]

(and, for hard classes, invoke the following constraints)

\[
\mu_i(x) \leq \nu_{iM} \quad \text{(for Class 1H objectives)}
\]

\[
\mu_i(x) \geq \nu_{iM} \quad \text{(for Class 2H objectives)}
\]

\[
\mu_i(x) = \nu_{iw} \quad \text{(for Class 3H objectives)}
\]

\[
\nu_{i5m} \leq \mu_i(x) \leq \nu_{i5M} \quad \text{(for Class 4H objectives)}
\]

\[
x_{j,\min} \leq x_j \leq x_{j,\max} \quad \text{(for design var. constraints)}
\]

where \( \nu_{iw}, \nu_{iM}, x_{j,\min}, \text{and} x_{j,\max} \) represent minimum and maximum values, and \( \nu_{iw} \) defines the equality constraints; the range limits are provided by the designer, and \( n_s \) is the number of soft objectives that the problem comprises. Note that the aggregate objective function only comprises class functions associated with soft objectives. The hard objectives are, by definition, constraints.

2.2 Rigidified inflatable structures

As stated earlier, Rigidified Inflatable Structures (RIS) are thin flexible membrane structures that are deployed by inflation. After deployment, the structures harden due to chemical or physical changes to the membrane, and no longer require pneumatic pressure to maintain structural stability. The resulting RIS consists of a structural skin that envelops a hollow cavity, and has the capability to withstand external loads (see Fig. 2).

The RIS membrane material typically consists of high-strength fibers that are embedded in an elastomeric matrix. Before deployment, the structure is compact, with a very small stowed volume and low weight, and the material has a low modulus of elasticity. The deployment process leaves the membrane system under tension due to the internal air pressure. The final structure then rigidifies due to an increase in the polymeric matrix stiffness and strength. The internal air used to inflate the structure is then vented, leaving a rigid walled structure.

3 Rigidified inflatable structures model

In this section, the generic testbed structure and its corresponding structural model are discussed. The associated finite element model is also presented.

3.1 RIS testbed configuration

The RIS selected for examination is a building of cylindrical configuration, with a floor area of 192 m\(^2\), a ceiling height of 3 m, and a wall thickness of 0.2 m to allow for acceptable thermal insulation properties. A cylindrical configuration offers ease of analysis, as it avoids modeling and numerical issues associated with connections between walls. Such issues are important and should be analyzed. However, the basic scope of this study is best addressed by using the cylindrical shape, which avoids unnecessary difficulties. Interestingly, we also note that in some cases (e.g., emergency housing), the end users might be specifically interested in such a simple, albeit unconventional, shape. Figure 2 depicts a partial cross-section of three bays of the RIS (top view).

In a previous study (Van Dessel et al. 2002b), analysis and optimization were performed on various tubular RIS wall systems with different bay sizes, resulting in the establishment of nominal bay sizes. The findings of that study are used here to set the width of each bay in the current model at approximately 0.35 m. With a diameter of 16 m, 144 bays are needed to build the entire structure.
Thus, the building comprises 144 contiguous bays, each of which is tubular in nature, with thin membranes on its four sides.

As can be seen in Fig. 2, two adjacent bays share a common membrane, which is straight and referred to as the side membrane. The side membranes have equal thicknesses, $t_2$. For each bay, the interior membrane has thickness $t_1$, while the exterior membrane has thickness $t_3$. Upon deployment, side membranes experience equal and opposite interior air pressures from each side, which allows them to maintain their nominal planar shapes. Under pneumatic pressure, the interior and exterior membranes inflate, resulting in their slightly curved shape.

To simplify the analysis and subsequent optimization, the structure is simplified by not modeling the roof explicitly. The roof load is accounted for by applying a distributed vertical load of 4000 N/m to the top of the cylindrical structure. This value was obtained by using the code-specified total roof load (for structural and snow loads), and subsequently dividing this load by the perimeter of the cylindrical wall. This approach is in keeping with the scope and objective of this study.

The other important consideration is wind load. A wind load of 1000 N/m$^2$ is applied to the structure, approximating a typical wind load at coastal locations (Building Officials & Code Administrators International Inc. 1996). The wind load is assumed to be unidirectional.

As discussed earlier regarding the scope of this study, we consider a continuous structure with no openings for windows or doors, in addition to an open overhead space instead of a roof structure. We believe these simplifications to have no significant impact in establishing the initial feasibility of RIS in the context of residential building. Recall that our key purpose here is to obtain feasible material properties, optimal dimensions, as well as to develop an efficient multiobjective structural optimization methodology using physical programming. The limitations discussed above, and others, should be the subject of subsequent work.

We discuss the finite element model and important modeling issues in the following subsection.

### 3.2 Finite element model

The complexity of the structure discussed above requires the use of finite element modeling and analysis. The finite element program Genesis (Vanderplaats R&D 2001) is used for modeling, analysis, and optimization – in conjunction with physical programming. Important modeling issues such as load cases and structural constraints are discussed, followed by details regarding the type of elements and mesh size used.

The RIS is modeled in its deployed and rigid state, and the subsequent analysis and optimization are performed on this structural model. The actual inflation process was not considered in this study.

**Applied loads**

To model the roof load, the total anticipated roof load is distributed equally among all the top nodes of the cylindrical wall model. This value is calculated based on the distributed roof load of 4000 N/m discussed above, and the circumference of the cylindrical structure; and is approximately 174 N/node with the mesh size used.

Figure 3 illustrates the idealized distribution of wind load on the building. The length of the arrows indicate the magnitude of the pressure applied in each region. As we consider the wind to be unidirectional, the wind load ($P = 1000$ N/m$^2$) is only applied to half of the structure. Generally, only the normal component of the wind, $P_n$, has an appreciable effect on the structure. The tangential component of the wind load, $P_t$, does not cause appreciable deflection on the structure, and can be neglected. It then follows that the normal component of the pressure is maximum, or equal to $P$, at the center of the semi-circle on which wind pressure is applied (see Fig. 4). The normal component is calculated as $P_n = P \cos \theta$, as shown in Fig. 4. At the center of the structure, $\theta = 0^\circ$, and at the sides, $\theta = 90^\circ$, where the wind load is tangential. Recall that no aerodynamic issues are addressed in this study.

We note that the value of the pressure varies continuously over the structure. However, without loss of generality, and in keeping with the characteristics of a discrete finite element analysis, we do not model the spatial distribution of the wind load continuously. The pressure is modeled as shown in Fig. 5. Instead of a continuous variation, the pressure is varied in steps. A different pres-
ure is applied for every 10 degrees of the semi-circle. This divides the half-surface into 17 regions, each with a different effective pressure value. The pressure value in each region is based on the angle at the center of each region.

**Structural constraints**

The structure is assumed to be connected to a foundation slab, with each bottom node pinned at the base of the wall. This means that the nodes on the bottom surface are constrained to not move along the \( x \), \( y \), and \( z \) axes, but are allowed to rotate about any of these axes. Figure 6 illustrates the pinned connection for one bay, and the resulting generic deformed shape. All degrees of freedom of all remaining nodes of the structure are unconstrained.

**Element type and mesh size**

Using Genesis, each bay is modeled with CQUAD4 plate elements, which are four-node quadrilateral elements with in-plane and bending stiffness. The appropriate dimension of the finite element mesh was determined by experimenting with several mesh sizes, and observing the sensitivity of analyses results for different mesh sizes. These experiments showed that the Buckling Load Factor (BLF) is particularly sensitive to mesh size. Note that buckling load factors are values that indicate the onset of buckling. Specifically, when the BLF is less than one, buckling has occurred. Under the same loading conditions, a coarse mesh results in a high buckling load factor, whereas a fine mesh results in a low and more accurate buckling load factor. An appropriate mesh size is one for which the buckling load factor value does not change appreciably with mesh size variation. This occurs at a rectangular mesh size of 0.3 m by 0.12 m for the model discussed above. With this mesh size, the model is relatively large in size, with 12,960 elements, 12,672 nodes and a total of 72,576 degrees of freedom.

The next section discusses the optimization problem formulation, the overall problem statement, and the optimization approach.

4 **Optimization problem formulation and approach**

This section describes the optimization problem formulation procedure for the structural model described in Sect. 3. The candidate RIS materials and their properties are described in the optimization approach.

4.1 **Optimization problem formulation**

In this study, we consider three materials for the RIS discussed above, and evaluate their respective performances to determine the most appropriate one. To perform this evaluation, we consider a set of design objectives, and their significance in the RIS context. In addition, we consider factors of typical importance in structural optimization problems. These include (i) stress, (ii) deflection, (iii) buckling, and (iv) cost. These quantities become constituent components of the aggregate objective function, and contribute to the constraints in the formulation of the optimization problem. Related discussions follow.

**Objectives**

*Stress*: We define the upper bound on the stress to be half the tensile strength of the material in question. Since we also wish to obtain a structural design that is minimally stressed, we also minimize stress in the RIS. Stress is therefore one of the design objectives we minimize as a component of the aggregate objective function.

*Deflection*: Guidelines for maximum allowable deflection in typical structures when subjected to loads are provided in various building codes (Building Officials & Code Administrators International Inc. 1996). Deflection constraints are incorporated into the optimization model by limiting deflection to \( l/120 \), where \( l \) is the height of the building, which is 3 m for the structure discussed in Sect. 3. This deflection criterion applies to wood structures with non-brittle finishes, and is used here because
of the unavailability of such criteria for RIS systems. In actuality, there may be a more lenient constraint on deflection for buildings constructed with the RIS materials considered in this study. Also, we note that minimal deflection is generally desired in any structure. Therefore, we minimize the deflection of the structure in our optimization model. The upper bound on deflection is defined according to the discussion above. The deflections are minimized only in the x and y directions, because preliminary analyses revealed that deflection is negligible in the z direction. This assumption was indeed confirmed in the examination of the final results.

**Buckling**: Buckling occurs when the load configuration applied on the structure reaches a certain critical magnitude (i.e., critical buckling load), resulting in structural failure. Buckling analysis is performed on a structure to reveal the buckling load factor (BLF) under the imposed load. The BLF is a measure of how close the applied load is to the critical buckling load. As stated earlier, a buckling load factor value of less than 1 indicates the occurrence of structural failure. In the optimization formulation, structural stability is ensured by constraining the BLF to be greater than 1. In addition, we let the BLF be a constituent component of the AOF in an effort to maximize it to the extent practical.

**Cost Analysis**: To establish the feasibility of different RIS material properties, careful consideration of the associated investments is required. The typical cost analysis tool for evaluating cost competitiveness is life cycle cost assessment. In the context of evaluating RIS life cycle cost, the latter is defined as the total investment required for the functional operation of the structure throughout its useful life. The cost is comprised of the initial cost, operation and maintenance cost, and financing cost. Initial costs are those associated with the design, manufacture, and installation of the structures. Operation costs include such costs as heating and cooling costs for the building, while repairs and renovations are categorized as maintenance costs. The financing cost is dependent upon the initial cost, and is higher with a larger initial cost.

For analysis purposes, the initial cost is taken as a representation of the overall life cycle cost. This is based on the assumption that the materials considered for RIS construction will have comparable operation and maintenance costs, as well as financing costs.

Specifically, initial costs (design, manufacturing, and installation) can be attributed to material, labor, equipment, and other miscellaneous costs. As stated earlier, an important objective of this research is to identify feasible material characteristics for RIS. As such, we focus on analyzing material costs. While different materials have different costs, they are assumed to undergo a general manufacturing process that results in typical labor and equipment costs for RIS system. Therefore, labor and equipment costs are not specifically addressed in this study; although these should be considered as the RIS technology matures, and data for estimating these costs becomes available. The total material cost is one design objective that we seek to minimize in the optimization process. Details about the costs of various materials are discussed in the optimization approach (Sect. 4.3).

**Selection of elements and nodes for responses**

Here we discuss some practical computational considerations that are invoked to facilitate the optimization process. The finite element model has approximately 12,960 elements and 12,672 nodes. Minimizing the stress in each element, and deflection at every node, would be unduly expensive computationally. Instead, we focus on the regions of maximum deflection and stress. A preliminary analysis is performed to reveal the elements that possess maximum stress, and the nodes with maximum deflection in the x and y directions (see Figs. 6 and 7). Figure 7 depicts the preliminary stress and deflection analysis results. The different color mappings indicate the different stress levels in each element. The minimum stress is marked on the figure with a small circle, while the maximum stress occurs near the bottom left of the figure. Also, the nodes with maximum deflection can be easily detected by examining the deformed

![Fig. 7 Maximum stress and deflection analysis – results](image-url)
structure. With these findings, stress is minimized only in the elements that have maximum stress. Three elements with maximum stress are chosen, one in each of the three membranes: interior, side, and exterior membranes (see Fig. 2). These stress quantities are denoted by \( \sigma_{t1} \), \( \sigma_{t2} \), and \( \sigma_{t3} \), respectively. The minimization of these three quantities is performed by making them part of the AOF, as three different design objectives.

We make the important observation that it is acceptable to minimize only these few stress values, rather than all the nodal stresses in the RIS, solely because of the nature of the parameterization of our mass distribution. In particular, we only have three membrane thicknesses for the cylindrical structure. Accordingly, since the loading is as indicated earlier, the stress variation is predictable; and will be generally of the same character as that illustrated in Fig. 7. These comments/assumptions apply to both stress and deflection; and are verified by examining the final results. We also note, parenthetically, that the choice of only three membrane thicknesses is motivated by practical manufacturing considerations.

In a fashion similar to the stress considerations above, deflection is minimized at only those nodes that have maximum deflection. Two nodes with maximum deflection in the \( x \) direction are selected, and the deflections at these nodes are denoted by \( \delta_{x1} \) and \( \delta_{x2} \). Two nodes with maximum deflection in the \( y \) direction are also selected, and the deflections at these nodes are denoted by \( \delta_{y1} \) and \( \delta_{y2} \). The deflections at these nodes are minimized as part of the aggregate objective function.

Thus, the stress and deflection objectives account for seven of the nine objectives. The other two design objectives are: cost of the entire structure, \( c \), and the buckling load factor, \( b \).

**Design variables**

The design objectives discussed above depend on the thickness of the RIS membranes. As a result, we consider three independent design variables: the interior membrane thickness \( (t_1) \), the side membrane thickness \( (t_2) \), and the exterior membrane thickness \( (t_3) \). The thickness values are restricted to lie between 0.1 and 3 mm, for practical manufacturing and deployment considerations.

**Constraints**

The elements and nodes selected for minimization are constrained to satisfy the allowable stress and deflection limits. Additionally, we constrain all other elements and nodes to satisfy the allowable stress and deflection limits.

### 4.2 Optimization problem statement

Based on the various design variables, design objectives, and constraints discussed in Sect. 4.1, the multiobjective optimization problem statement is presented as follows:

\[
\begin{bmatrix}
  b \\
  c \\
  \sigma_{t1} \\
  \sigma_{t2} \\
  \sigma_{t3} \\
  \delta_{x1} \\
  \delta_{x2} \\
  \delta_{y1} \\
  \delta_{y2}
\end{bmatrix}
\]

subject to

\[
\begin{align*}
\max \sigma & \leq \frac{\sigma_{tensile}}{2} \\
\delta_x & \leq \left( \frac{l}{120} \right) \\
\delta_y & \leq \left( \frac{l}{120} \right) \\
\end{align*}
\]

where \( b \) is the buckling load factor, \( c \) is the total material cost, \( \sigma_{t1} \), \( \sigma_{t2} \), and \( \sigma_{t3} \) are the maximum stresses in the interior membrane, side membrane, and exterior membrane, respectively, and \( \delta_{x1} \), \( \delta_{x2} \), \( \delta_{y1} \), and \( \delta_{y2} \) are the maximum deflections in the \( x \) and \( y \) directions. (3) constrains the stresses of all elements to be less than half the material tensile strength, \( \sigma_{tensile} \). (4) and (5) constrain the deflections of all nodes in the \( x \) and \( y \) directions to be less than the maximum deflection enforced by building codes (\( l = 3 \text{ m} \)) (Building Officials & Code Administrators International Inc. 1996). The last constraint enforces the upper and lower bounds of the allowable membrane thicknesses.

Additionally, the following equations apply:

\[
[K - bK_y] \Phi = 0 \tag{7}
\]

\[
c = c_u V \tag{8}
\]

\[
p = Kd. \tag{9}
\]

The buckling analysis performed in Genesis using the finite element method is governed by (7), where \( K \) is the system stiffness matrix, \( K_y \) is the system geometric stiffness matrix, and \( \Phi \) is the buckling mode shape (Vanderplaats R&D 2001). The cost of material per unit volume is \( c_u \), and \( V \) is the total material volume of the structure. (9) enforces static equilibrium, where \( d \) is the displacement vector and \( p \) is the load vector. These equations, evaluated by Genesis, effectively constitute additional constraints in the optimization formulation.
4.3 Optimization approach

Here we discuss the various design cases considered, and the optimization methodology.

Materials

Three design options that correspond to three different material types are considered. The materials considered are all polymers, non-reinforced and reinforced. Equivalent isotropic material properties are assumed. The broad selection of the materials to be analyzed is guided by desired material properties (e.g., strength, cost, density). This selection is not intended to specifically point to a precise given material, but instead to a class of materials. Table 1 lists the general properties of the examined materials, as listed by Callister, Jr. (1999). The materials in Class 1 are non-reinforced polymers; the materials in Class 2 are polymers lightly reinforced with randomly oriented, discontinuous E-glass fibers; and the materials in Class 3 are polymers reinforced with uniformly oriented, continuous E-glass fibers. Approximate cost data for each material class is obtained from several sources (RS Means Company Inc. 1998; Accurate Plastics Inc. 2002; Callister, Jr. 1999), for materials that exhibit similar properties as those listed in Table 1. For material Class 1, polyvinyl chloride (PVC) single ply membrane (which is typically used for roofing) is chosen for cost data. The properties of rigid PVC are chosen to reflect the mechanical characteristics for a Class 1 material. A typical thermoset sheet molding compound (SMC), which is an E-glass reinforced polyester, is selected to represent Class 2 material costs. Class 3 material costs are represented by epoxy prepreg reinforced with continuous E-glass fibers.

Optimization methodology – Physical Programming

The structural optimization problem discussed above has strongly conflicting objectives, such as minimizing cost and deflection, and minimizing cost and stress. Also, the values of the design objectives (as revealed in the preliminary analysis) are poorly scaled. The deflection is on the order of 0.01, while the stress is on the order of 10 million. In addition, there is no guarantee that the associated Pareto frontier is convex – making some multiobjective optimization approaches inappropriate. We are also interested in adequately expressing our present state of knowledge of our preferences in the formulation of the optimization problem. As discussed earlier, this desire to effectively address (i) the multiobjective nature of the problem, (ii) the poor scaling of the problem, and (iii) the ability to express our preference explicitly, all point to our choice of physical programming (Messac 1996) as the basis of our optimization problem formulation. It is important to keep in mind that our preferences with respect to the individual objectives may evolve during the design process. As we obtain various results, and uncover what is possible and what is not, we are likely to refine our stated preferences; and we do so using physically meaningful desirability values for the design objectives. As discussed in several publications, the optimum solutions are not unduly sensitive to the preference values chosen. Next, we discuss the articulation of preferences.

Preferences

Preferences regarding each design objective are expressed according to the six preference ranges (see Sect. 2.1) by capitalizing on the Decision Maker’s (DM) design knowledge and the structural design requirements. To formulate the preferences, the DM may combine a variety of information, such as results from preliminary analysis, desirable outcome values, as well as the availability of a full or partial trade-off matrix, which can be obtained by conducting a set of optimization runs.

To formulate preferences, the DM needs to choose five desirability values that indicate her/his preference for each design objective’s final value, with respect to different ranges of desirability. For reasons having to do with logical expression of preferences, it is advisable to let each range be equal to or smaller than its neighboring better range. That is, the Tolerable range is equal to or smaller than the Desirable range, and so on.

In choosing the preference values, several considerations can be taken into account. First, hard constraints (for example) can be incorporated into the preferences formulation as the upper limit to the Highly Undesirable range, \( v_5 \), for Class 1S. Next, trial optimization runs can be conducted to obtain information on the trade-offs between design objectives. Alternatively, information from Decision Makers or domain experts can be used.

In the current structural optimization problem, there are hard constraints on the acceptable buckling load factor and on deflection. Structures cannot have a buckling load factor of less than 1, as this means the structure would fail under the specified loading condition. The maximum deflection limit is placed to satisfy building code requirements, according to the maximum acceptable deflection of wood structures with non-brittle finishes. Violating the maximum acceptable deflection, however, does not necessarily affect the structure’s integrity, as is the case with violating the buckling constraint. The hard constraints on buckling and deflection are incorporated as \( v_5 \) in the preference values, as discussed earlier (i.e., Unacceptable range). The preferences for the design objectives of this structural optimization problem are shown in Table 2. We note parenthetically that we have used the

<table>
<thead>
<tr>
<th>Material Class</th>
<th>Modulus of Elasticity (GPa)</th>
<th>Tensile Strength (MPa)</th>
<th>Density (kg/m³)</th>
<th>Cost ($/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.0</td>
<td>30.0</td>
<td>1400</td>
<td>2.50</td>
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<tr>
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<td>100.0</td>
<td>1900</td>
<td>11.00</td>
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<td>3</td>
<td>30.0</td>
<td>300.0</td>
<td>2100</td>
<td>22.00</td>
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</table>
Table 2 Decision Maker preference table

<table>
<thead>
<tr>
<th>Metrics</th>
<th>Unit</th>
<th>Class</th>
<th>$ν_{15}$</th>
<th>$ν_{14}$</th>
<th>$ν_{13}$</th>
<th>$ν_{12}$</th>
<th>$ν_{11}$</th>
<th>$ν_{10}$</th>
<th>$ν_{19}$</th>
<th>$ν_{18}$</th>
<th>$ν_{17}$</th>
<th>$ν_{16}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>2S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>$$</td>
<td>1S</td>
<td>20,000</td>
<td>18,000</td>
<td>14,000</td>
<td>8,000</td>
<td>200</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$δ_{x1}$</td>
<td>cm</td>
<td>1S</td>
<td>2.5</td>
<td>2.3</td>
<td>2.0</td>
<td>1.3</td>
<td>0.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$δ_{x2}$</td>
<td>cm</td>
<td>1S</td>
<td>2.5</td>
<td>2.3</td>
<td>2.0</td>
<td>1.3</td>
<td>0.3</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$δ_{y1}$</td>
<td>cm</td>
<td>1S</td>
<td>2.5</td>
<td>2.3</td>
<td>2.0</td>
<td>1.3</td>
<td>0.3</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$δ_{y2}$</td>
<td>cm</td>
<td>1S</td>
<td>2.5</td>
<td>2.3</td>
<td>2.0</td>
<td>1.3</td>
<td>0.3</td>
<td></td>
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<td>3</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>$σ_{\text{mat2}}$</td>
<td>MPa</td>
<td>1S</td>
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<td>30</td>
<td>10</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>$σ_{\text{mat3}}$</td>
<td>MPa</td>
<td>1S</td>
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<td>140</td>
<td>110</td>
<td>75</td>
<td>70</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

HU = highly undesirable, U = undesirable, T = tolerable, D = desirable, HD = highly desirable

absolute values of the maximum deflection in the expression of preference in the table, purely for convenience. This has no consequence because these values are always either positive or negative; and will not take on the non-smooth zero value – which might cause computational difficulties.

The preferences for cost in this optimization process are of special note. The range of values is particularly wide, ranging from $200 to $20,000. In this particular case we allow a wide spectrum of possibilities, from the very inexpensive to the very expensive. Another reason for the wide range is the large difference between the unit costs of the three materials examined, ranging from 2.50 $/kg to approximately ten times that, at 22 $/kg (see Table 1).

The preferences for the buckling load factor, cost, and deflections are the same for all three design cases, with different materials. However, the preferences for stress vary according to the tensile strength of each material. Since the amount of stress each material can sustain depends on its tensile strength, the stress preferences are expressed accordingly.

Interaction between Genesis and Physical Programming

Similar to other codes, Genesis is limited in the variety of multiobjective optimization methods that it can support. Therefore, the calculations for the AOF are performed externally by Physical Programming, and the calculated value of the AOF for the current design is passed to Genesis during the optimization process.

The optimization approach consists of two stages: (i) preliminary analysis to obtain basic characteristics of the structure at hand, and (ii) articulation of preferences followed by optimization (see Fig. 8). In the first stage, the structural model that corresponds to the RIS building configuration under consideration is constructed. Preliminary structural analysis is then performed to determine the elements with the highest stress for each of the three membranes, two nodes with the highest deflection in the x direction, and two nodes with the highest deflection in the y direction. The three elements and four nodes are the locations at which stress and deflection minimization is performed, as discussed earlier. The optimization problem is then formulated by using the information from preliminary analysis; in part, to articulate preferences.

In the next stage, as outlined in the dashed box of Fig. 8, Genesis and Physical Programming are used in tandem to perform the optimization. During one optimization iteration, Genesis performs structural analysis and optimization and generates responses (e.g., stress, buckling load factor, volume, etc.). These values are then passed as design objectives to Physical Programming, where the aggregate objective function (AOF) value is computed. The resulting AOF value is passed back to Genesis, where it is minimized. The design cycle is then repeated until an optimal solution is found.

Optimization results of this large-scale structural problems are given in the next section.

5 Results and discussion

Optimizations are performed for the three design cases. One set of optimum results is obtained for each material.
Table 3 Optimization results

<table>
<thead>
<tr>
<th>Material Class</th>
<th>Thickness (mm)</th>
<th>Buckling Load Factor</th>
<th>Cost ($)</th>
<th>Stress (MPa)</th>
<th>Deflection (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t₁</td>
<td>t₂</td>
<td>t₃</td>
<td>σ₁₁</td>
<td>σ₁₂</td>
</tr>
<tr>
<td>1</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
<td>2.22</td>
<td>4130</td>
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<tr>
<td>2</td>
<td>1.73</td>
<td>1.24</td>
<td>1.77</td>
<td>1.34</td>
<td>13524</td>
</tr>
<tr>
<td>3</td>
<td>0.78</td>
<td>1.14</td>
<td>0.91</td>
<td>1.21</td>
<td>16520</td>
</tr>
</tbody>
</table>

* Infeasible

Table 4 Relaxed preference on displacement for Material 1

<table>
<thead>
<tr>
<th>Metrics</th>
<th>Unit</th>
<th>Class</th>
<th>νₓ₁</th>
<th>νₓ₄</th>
<th>νᵧ₁</th>
<th>νᵧ₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>δₓ₁</td>
<td>cm</td>
<td>1S</td>
<td>5</td>
<td>4.5</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>δₓ₂</td>
<td>cm</td>
<td>1S</td>
<td>5</td>
<td>4.5</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>δᵧ₁</td>
<td>cm</td>
<td>1S</td>
<td>5</td>
<td>4.5</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>δᵧ₂</td>
<td>cm</td>
<td>1S</td>
<td>5</td>
<td>4.5</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

HU = highly undes., U = undesirable, T = tolerable, D = desirable, HD = highly des.

Optimum values of design objectives and design variables are recorded in Table 3. We discuss the performance of each material below.

Material Class 1

Table 5 provides a summary of the performance of the materials with respect to the DM’s preferences. In this table, we note that the Class 1 material performs comparably to the other material classes in all but one design objective, the y deflection. This material has a lower modulus of elasticity than the other material classes, which limits its ability to sustain the given wind load. To satisfy the deflection constraint on the overall structure (l/120), the design requires a membrane thickness of more than 3 mm. However, there are critical practical considerations associated with thick membranes. Accordingly, a maximum membrane thickness is enforced. At the maximum membrane thickness allowed (3 mm), Material 1 violates the y deflection constraint by approximately 1 cm.

This is an unfortunate drawback because, as can be seen in Fig. 9, Material 1 performs rather well for other design objectives, such as cost and buckling. With this performance in mind, this material may conditionally be a feasible material for RIS construction, if one is more tolerant of deflection. Two issues should be kept in mind when considering this material: (i) The constraint on deflection originates from building codes for wood structures, whose ability to safely deform is lower than that of a RIS system using Material 1. This implies that there may be more allowance on deflection for RIS constructed with Material 1. (ii) The ultimate practical use of the RIS building also needs to be considered. With the relatively low cost of the building, and the ease of construction and transportation of RIS systems, they may be used for disaster relief and/or low-cost residence. For this type of usage, there may be more tolerance on structural deflection, and the findings gathered here may position Material 1 as appropriate and perhaps optimal for such RIS buildings.

To examine this proposition, we relaxed the deflection preferences for Material 1 to l/60 (see Table 4) to

Table 5 Result summary

<table>
<thead>
<tr>
<th>Material</th>
<th>Material 1</th>
<th>Material 2</th>
<th>Material 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buckling</td>
<td>HD</td>
<td>T</td>
<td>U</td>
</tr>
<tr>
<td>Cost</td>
<td>D</td>
<td>T</td>
<td>U</td>
</tr>
<tr>
<td>Stress</td>
<td>D</td>
<td>HD</td>
<td>HD</td>
</tr>
<tr>
<td>x Deflection</td>
<td>T</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>y Deflection</td>
<td>Violated</td>
<td>T</td>
<td>D</td>
</tr>
</tbody>
</table>

Fig. 9 Material Class 2 and 3 comparison
make our deflection Tolerable, and performed the optimization based on this new set of preferences. The results of this new optimization indicate that the balance between the objectives did not appreciably change, but with the deflection now within a feasible range. This confirms that Material 1 could be an appropriate option for use in RIS.

**Material Class 2 and 3**

The other two materials, Class 2 and 3, satisfy all the hard constraints, and are unconditionally feasible RIS materials. A comparison of material Class 2 and 3 is illustrated in Table 5 and Fig. 9. Figure 9 compares four factors: buckling, cost, (maximum) stress, and (maximum) deflection (in either the x or y direction). In this figure, the comparison of maximum stress is based on the different preference sets of each material class, unlike the other objectives – which have the same preferences across the different materials.

The values of the stress for both materials lie in the Highly Desirable region, and although Material 3 results in a slightly better performance, both still lie in a region in which the actual stresses are significantly less than the allowable stress levels. The maximum deflection for Material 3 is marginally better than that of Material 2; however both are within the Tolerable or the Desirable range. For this reason, and the previously mentioned fact that the deflection constraint is based on wood structures, we may conclude that both of these objectives can be adequately dealt with. On the other hand, buckling and cost are two critical design objectives, since the associated values for both materials are in or the Undesirable range. The value for the buckling load factor, for example, is extremely important, as it affects structural integrity. Keeping all these factors in mind, Material 2 appears to be a better RIS material. Cost is another important design objective, as it lies in the Tolerable range for Material 2, and lies in the Undesirable range for Material 3. For the purpose of obtaining a low-cost material, Material 2 appears to be the better alternative. Although Material 3 is characterized by superior properties in terms of modulus of elasticity and strength, its increase in cost may not be justifiable.

**Material selection**

In light of the previous findings, among the three material classes examined, Material 2 yields the best overall performance. Alternatively, Material 1 is a possible RIS material for use when there is a degree of leniency on the maximum allowable deflection.

An important observation here is that, for all three materials, stress is not a limiting factor throughout the optimization process. Even for the weakest material ($\sigma_{\text{tensile}} = 30 \text{ MPa}$), the maximum stress on the structure is minimal. Therefore, during the process of material selection for the RIS system examined in this study, a high tensile strength is not a driving factor.

**6 Concluding remarks**

In this paper, a multiobjective structural optimization approach based on Physical Programming (PP) is presented, and used to optimize large-scale Rigidified Inflatable Structures (RIS). The optimization approach developed provides a new effective tool for structural optimization in the RIS context. The incorporation of PP into the optimization process enables an unambiguous formulation of an aggregate objective function that facilitates effective multiobjective structural optimization for large-scale problems.

This research initiates a meaningful step towards the infusion of RIS into residential construction by establishing feasible combinations of RIS material properties and structural configurations to be used in residential construction. Further study may address issues that have been simplified in the course of this work, such as the inclusion of a roof/floor structure and windows and doors, as well as the study of alternative RIS wall configurations.

In addition, this study brings the structural optimization community and the architecture and civil engineering communities a step closer together – in the context of innovative building technologies. Indeed, this research was a multidisciplinary effort involving architects and engineers.

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